

Information, information processing and gravity

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I discuss fundamental limits placed on information and information processing by gravity. Such limits arise because both information and its processing require energy, while gravitational collapse (formation of a horizon or black hole) restricts the amount of energy allowed in a finite region. Specifically, I use a criterion for gravitational collapse called the hoop conjecture. Once the hoop conjecture is assumed a number of results can be obtained directly: the existence of a fundamental uncertainty in spatial distance of order the Planck length, bounds on information (entropy) in a finite region, and a bound on the rate of information processing in a finite region. In the final section I discuss some cosmological issues, related to the total amount of information in the universe, and note that almost all detailed aspects of the late universe are determined by the randomness of quantum outcomes. This paper is based on a talk presented at a 2007 Bellairs Research Institute (McGill University) workshop on black holes and quantum information.

I. INTRODUCTION

This paper is based on a talk presented at a workshop on black holes and quantum information (Bellairs Research Institute of McGill University, Barbados). Most of the participants were quantum information theorists, so I attempted to keep the technical details concerning general relativity or particle physics at a minimum. I tried to summarize, in the most physical and intuitive way, how gravity enforces some surprising constraints on information and information processing. From a practical perspective, due to the feebleness of the gravitational force, all of the limits deduced are incredibly weak. Our technologies are nowhere near saturating them, and they are of much greater interest to theoreticians than experimentalists or engineers. Nevertheless, they are fundamental in nature, and, depending as they do both on quantum mechanics and general relativity, may offer a view into the properties of quantum gravity.

In the discussion that follows, gravitational collapse will be our crude but powerful probe of gravitational physics. Complete gravitational collapse leads to the formation of black holes and causal horizons. Gravity is a long range force that, as far as we know, *cannot be screened*. In this respect, it is fundamentally different from gauge forces, such as the strong and electroweak interactions, and it is precisely this difference that allows for dramatic phenomena like complete collapse.

We use Planck units throughout, in which the speed of light, Planck's constant and the Planck mass (equivalently, Newton's constant) are unity. In our expressions, any energy or mass is therefore measured in units of 10^{19} GeV (about 10^{-5} grams), and any length is measured in units of the Planck length, or about 10^{-35} meters.

II. GRAVITATIONAL COLLAPSE

Ideally, one would like to know precisely what subset of all possible physical initial data results in gravitational collapse and the formation of a black hole. This is obviously a difficult problem and it is currently unsolved. Schoen and Yau [1] proved a celebrated result requiring the existence of a closed trapped surface if the *minimum* density in a region is sufficiently high. However, this result fails to be useful if the energy of the initial configuration is distributed in a very nonuniform manner.

Note that results for black hole or horizon formation typically require both the assumption of the null or weak energy condition and of cosmic censorship [2]. Under those assumptions a closed trapped surface can be shown to result in a singularity (using the Raychaudhuri equation and assuming the energy conditions hold), and cosmic censorship requires a horizon to conceal the singularity from asymptotic observers.

In our analysis we will use the hoop conjecture, due to Kip Thorne [3] as a criterion for gravitational collapse. It states that a system of total energy E , if confined to a sphere of radius $R < \eta E$ (η is a coefficient of order one, which we neglect below), must inevitably evolve into a black hole. The condition $R < E$ is readily motivated by the Schwarzschild radius $R_s = 2M$. This conjecture has been confirmed in astrophysically-motivated numerical simulations, and has been placed on even stronger footing by recent results on black hole formation from relativistic particle collisions [4]. These results show that, even in the case when all of the energy E is provided by the kinetic energy of two highly boosted particles, a black hole forms whenever the particles pass within a distance of order E of each other (see Fig. 1). Two particle collisions had seemed the most likely to provide a counterexample to the conjecture, since the considerable kinetic energy of each particle might have allowed escape from gravitational collapse.

One can think of the hoop conjecture as requiring that the average energy density of an object of size R be less

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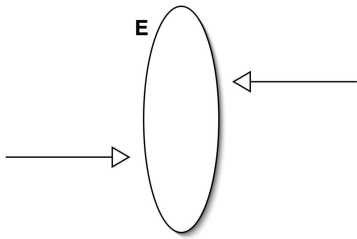


FIG. 1: The hoop conjecture applied to two relativistic particles.

than R^{-2} in order not to collapse to a black hole. Thus, large objects which are not black holes must be less and less dense. For example, a sufficiently large object with only the density of water will eventually form a black hole!

III. MINIMAL LENGTH

In this section we deduce a fundamental limit on our ability to measure a distance [5, 6, 7, 8]. The results suggest that spacetime may ultimately have a discrete structure. At the end of the section we discuss the implications for quantum information and the ultimate Hilbert space of quantum mechanics.

From the hoop conjecture (HC) and the uncertainty principle, we immediately deduce the existence of a *minimum ball* of size l_P . Consider a particle of energy E which is not already a black hole. Its size r must satisfy

$$r \gtrsim \max[1/E, E] \quad , \quad (1)$$

where $\lambda_C \sim 1/E$ is its Compton wavelength and E arises from the hoop conjecture. Minimization with respect to E results in r of order unity in Planck units, or $r \sim l_P$ [9]. If the particle *is* a black hole, then its radius grows with mass: $r \sim E \sim 1/\lambda_C$. This relationship suggests that an experiment designed (in the absence of gravity) to measure a short distance $l \ll l_P$ will (in the presence of gravity) only be sensitive to distances $1/l$. This is the classical counterpart to T-duality in string theory [10].

It is possible that quantum gravitational corrections modify the relation between E and R in the HC. However, if E is much larger than the Planck mass, and R much larger than l_P , we expect semiclassical considerations to be reliable. (Indeed, in two particle collisions with center of mass energy much larger than the Planck mass the black holes produced are semiclassical.) This means that the existence of a minimum ball of size much smaller than l_P does *not* depend on quantum gravity - the energy required to confine a particle to a region of size much smaller than l_P would produce a large, semiclassical black hole.

Before proceeding further, we give a concrete model of minimum length that will be useful later. Let the position operator \hat{x} have discrete eigenvalues $\{x_i\}$, with

the separation between eigenvalues either of order l_P or smaller. (For regularly distributed eigenvalues with a constant separation, this would be equivalent to a spatial lattice.) We do not mean to imply that nature implements minimum length in this particular fashion - most likely, the physical mechanism is more complicated, and may involve, for example, spacetime foam or strings. However, our concrete formulation lends itself to detailed analysis. We show below that this formulation cannot be excluded by any gedanken experiment, which is strong evidence for the existence of a minimum length.

Quantization of position does not by itself imply quantization of momentum. Conversely, a continuous spectrum of momentum does not imply a continuous spectrum of position. In a formulation of quantum mechanics on a regular spatial lattice, with spacing a and size L , the momentum operator has eigenvalues which are spaced by $1/L$. In the infinite volume limit the momentum operator can have continuous eigenvalues even if the spatial lattice spacing is kept fixed. This means that the displacement operator

$$\hat{x}(t) - \hat{x}(0) = \hat{p}(0) \frac{t}{M} \quad (2)$$

does not necessarily have discrete eigenvalues (the right hand side of (2) assumes free evolution; we use the Heisenberg picture throughout). Since the time evolution operator is unitary the eigenvalues of $\hat{x}(t)$ are the same as $\hat{x}(0)$. Importantly though, the spectrum of $\hat{x}(0)$ (or $\hat{x}(t)$) is completely unrelated to the spectrum of the $\hat{p}(0)$, even though they are related by (2) [11]. Consequently, we stress that *a measurement of the displacement is a measurement of the spectrum of $\hat{p}(0)$ (for free evolution) and does not provide information on the spectrum of \hat{x}* . A measurement of arbitrarily small displacement (2) does not exclude our model of minimum length. To exclude it, one would have to measure a position eigenvalue x and a nearby eigenvalue x' , with $|x - x'| \ll l_P$.

Many minimum length arguments (involving, e.g., a microscope, scattering experiment or even Wigner's clock [6]) are obviated by the simple observation of the minimum ball. However, the existence of a minimum ball does not by itself preclude the localization of a macroscopic object to very high precision. Hence, one might attempt to measure the spectrum of $\hat{x}(0)$ through a time of flight experiment in which wavepackets of primitive probes are bounced off of well-localised macroscopic objects. Disregarding gravitational effects, the discrete spectrum of $\hat{x}(0)$ is in principle obtainable this way. But, detecting the discreteness of $\hat{x}(0)$ requires wavelengths comparable to the eigenvalue spacing. For eigenvalue spacing comparable or smaller than l_P , gravitational effects cannot be ignored, because the process produces minimal balls (black holes) of size l_P or larger. This suggests a direct measurement of the position spectrum to accuracy better than l_P is not possible. The failure here is due to the use of probes with very short wavelength.

A different class of instrument - the interferometer

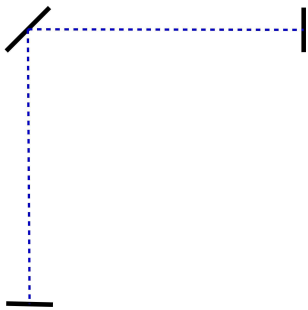


FIG. 2: An interferometer can be sensitive to path length differences much smaller than the wavelength of light used.

(Fig. 3) - is capable of measuring distances much smaller than the size of any of its sub-components [12]. An interferometer can measure a distance

$$\Delta x \sim \frac{\lambda}{b\sqrt{N}} \sim \frac{L}{\tau\sqrt{N}\nu} , \quad (3)$$

where $\lambda = 1/\nu$ is the wavelength of light used, L is the length of each arm, τ the time duration of the measurement, and N the number of photons. More precisely, Δx is the change over the duration of the measurement in the relative path lengths of the two arms of the interferometer. $b = \tau/L$ is the number of bounces over which the phase difference builds, so (3) can also be written as

$$\Delta\Phi = \frac{b\Delta x}{\lambda} \sim \frac{1}{\sqrt{N}} , \quad (4)$$

which expresses saturation of the quantum mechanical uncertainty relationship between the phase and number operators of a coherent state.

From (3) it appears that Δx can be made arbitrarily small relative to λ by, e.g., taking the number of bounces to infinity. Were this the case, we would have an experiment that, while still using a wavelength λ much larger than l_P , could measure a distance less than l_P along one direction, albeit at the cost of making the measured object (e.g., a gravity wave) large in the time direction. This would contradict the existence of a *minimum interval*, though not a minimum ball in spacetime. (Another limit which increases the accuracy of the interferometer is to take the number of photons N to infinity, but this is more directly constrained by gravitational collapse. Either limit is ultimately bounded by the argument discussed below.)

A constraint which prevents an arbitrarily accurate measurement of Δx by an interferometer arises due to the Standard Quantum Limit (SQL) and gravitational collapse. The SQL [13] is derived from the uncertainty principle (we give the derivation below; it is not specific to interferometers, although see [14]) and requires that

$$\Delta x \geq \sqrt{\frac{t}{2M}} , \quad (5)$$

where t is the time over which the measurement occurs and M the mass of the object whose position is measured. In order to push Δx below l_P , we take b and t to be large. But from (5) this requires that M be large as well. In order to avoid gravitational collapse, the size R of our measuring device must also grow such that $R > M$. However, by causality R cannot exceed t . Any component of the device a distance greater than t away cannot affect the measurement, hence we should not consider it part of the device. These considerations can be summarized in the inequalities

$$t > R > M . \quad (6)$$

Combined with the SQL (5), they require $\Delta x > 1$ in Planck units, or

$$\Delta x > l_P . \quad (7)$$

(Again, we neglect factors of order one.)

Notice that the considerations leading to (5), (6) and (7) were in no way specific to an interferometer, and hence are *device independent*. We repeat: no device subject to the SQL, gravity and causality can exclude the quantization of position on distances less than the Planck length.

It is important to emphasize that we are deducing a minimum length which is parametrically of order l_P , but may be larger or smaller by a numerical factor. This point is relevant to the question of whether an experimenter might be able to transmit the result of the measurement before the formation of a closed trapped surface, which prevents the escape of any signal. If we decrease the minimum length by a numerical factor, the inequality (5) requires $M \gg R$, so we force the experimenter to work from deep inside an apparatus which has far exceeded the criterion for gravitational collapse (i.e., it is much denser than a black hole of the same size R as the apparatus). For such an apparatus a horizon will already exist before the measurement begins. The radius of the horizon, which is of order M , is very large compared to R , so that no signal can escape.

We now give the derivation of the Standard Quantum Limit. Consider the Heisenberg operators for position $\hat{x}(t)$ and momentum $\hat{p}(t)$ and recall the standard inequality

$$(\Delta A)^2(\Delta B)^2 \geq -\frac{1}{4}(\langle[\hat{A}, \hat{B}]\rangle)^2 . \quad (8)$$

Suppose that the position of a *free* test mass is measured at time $t = 0$ and *again* at a later time. The position operator at a later time t is

$$\hat{x}(t) = \hat{x}(0) + \hat{p}(0)\frac{t}{M} . \quad (9)$$

The commutator between the position operators at $t = 0$ and t is

$$[\hat{x}(0), \hat{x}(t)] = i\frac{t}{M} , \quad (10)$$

so using (8) we have

$$|\Delta x(0)| |\Delta x(t)| \geq \frac{t}{2M} . \quad (11)$$

So, at least one of the uncertainties $\Delta x(0)$ or $\Delta x(t)$ must be larger than of order $\sqrt{t/M}$. As a measurement of the discreteness of $\hat{x}(0)$ requires *two* position measurements, it is limited by the greater of $\Delta x(0)$ or $\Delta x(t)$, that is, by $\sqrt{t/M}$.

What are the consequences of a minimum length? In a discrete spacetime there need not be any continuous degrees of freedom, and the number of degrees of freedom in a fixed volume is finite. Further, one can show that discretization of spacetime naturally suggests discretization of Hilbert space itself [15]. Specifically, in a universe with a minimal length (for example, due to quantum gravity), no experiment can exclude the possibility that Hilbert space is discrete.

IV. ENTROPY BOUNDS

In this section we describe two entropy bounds arising from gravitational collapse. These bounds limit the number of degrees of freedom in a region of size R , or equivalently the amount of information in any system of fixed size.

The first bound uses the area–entropy relation for black holes. Black holes radiate [16] and have entropy: $S = A/4$ [17]. The nature of this entropy is one of the great mysteries of modern physics, especially due to its non-extensive nature: it scales as the area of the black hole (in Planck units), rather than its volume. This peculiar property has led to the holographic conjecture [18, 19] proposing that the number of degrees of freedom in any region of our universe grows only as the area of its boundary. (See [20] for a review and discussion of covariant generalizations of this idea, and [21] for a general discussion of how area bounds arise in gravitating systems.) The AdS/CFT correspondence [22] is an explicit realization of holography.

The entropy of a thermodynamic system is the logarithm of the number of the available microstates of the system, subject to some macroscopic constraints such as fixed total energy. In certain string theory black holes, these states have been counted explicitly [23, 24].

Consider a system of size R and total energy E (e.g., the green blob in Fig. 3), which is not a black hole ($E < R$). Now imagine a spherical shell of energy $R - E$ approaching the system at the speed of light. By causality, the system is unaffected by the shell until the combination of the two already satisfy the hoop conjecture. The combined system must evolve into a black hole with entropy $A/4$. By the second law of thermodynamics, this final entropy is larger than that of the initial system. Since we made no particular assumptions about the initial system, we deduce that ordinary (non-collapsed) physical systems have entropy less than their surface area

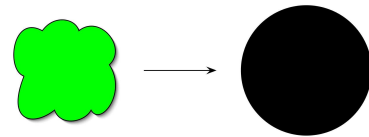


FIG. 3: A system collapses to form a black hole. By the second law of thermodynamics, its original entropy is bounded above by that of the black hole.

in Planck units. This is quite a counterintuitive result, since in familiar (non-gravitating) systems entropy is typically extensive.

The second bound, obtained by 't Hooft [25], shows that if one excludes states from the Hilbert space whose energies are so large that they would have already caused gravitational collapse, one obtains $S = \ln N < A^{3/4}$, where N is the number of degrees of freedom and A the surface area. To deduce this result, 't Hooft replaces the system under study with a thermal one. This is justified because, in the large volume limit, the entropy of a system with constant total energy E (i.e., the logarithm of the phase space volume of a microcanonical ensemble) is given to high accuracy by that of a canonical ensemble whose temperature has been adjusted so that the average energies of the two ensembles are the same. (This is a standard, and central, result in statistical mechanics.)

Given a thermal region of radius R and temperature T , we have $S \sim T^3 R^3$ and $E \sim T^4 R^3$. Requiring $E < R$ then implies $T \sim R^{-1/2}$ and $S < R^{3/2} \sim A^{3/4}$. We stress that the thermal replacement is just a calculational trick: temperature plays no role in the results, which can also be obtained by direct counting.

In [26], it was shown that imposing the condition $\text{Tr}[\rho H] < R$ on a density matrix ρ implies a similar bound $S_{\text{vN}} < A^{3/4}$ on the von Neumann entropy $S_{\text{vN}} = -\text{Tr} \rho \ln \rho$. For ρ a pure state the result reduces to the previous Hilbert space counting.

We note that these bounds are more restrictive than the bound obtained from black hole entropy: $S < A/4$. One can interpret this discrepancy as a consequence of higher entropy density of gravitational degrees of freedom relative to ordinary matter [27].

The consequences of these bounds are rather striking: they suggest that gravitating systems in d dimensions contain only as much information as analogous, but non-gravitating, systems in $d - 1$ dimensions. A concrete realization of this is the AdS/CFT duality in string theory [22].

V. BOUND ON RATE OF INFORMATION PROCESSING

In this section we derive an upper bound on the rate at which a device can process information [28]. We define this rate as the number of logical operations per unit time, denoted as the ops rate \mathcal{R} . The operations in ques-

tion can be those of either classical or quantum computers. The basis of our result can be stated very simply: information processing requires energy, and general relativity limits the energy density of any object that does not collapse to a black hole. Replacing *information processing* by *information* in the previous sentence leads to holography or black hole entropy bounds, a connection we will explore further below. For related work on fundamental physical limits to computation, see [29] and [30].

Our result is easily deduced using the Margolus–Levitin (ML) theorem [31] from quantum mechanics, and the hoop conjecture.

The Margolus–Levitin theorem states that a quantum system with average energy ϵ requires at least $\Delta t > \epsilon^{-1}$ to evolve into an orthogonal (distinguishable) state. It is easy to provide a heuristic justification of this result. For an energy eigenstate of energy E , E^{-1} is the time required for its phase to change by order one. In a two state system the energy level splitting E is at most of order the average energy of the two levels. Then, the usual energy-time uncertainty principle suggests that the system cannot be made to undergo a controlled quantum jump on timescales much less than E^{-1} , as this would introduce energy larger than the splitting into the system.

Consider a device of size R and volume $V \sim R^3$, comprised of n individual components [32] of average energy ϵ . Then, the ML theorem gives an upper bound on the total number of operations per unit time

$$\mathcal{R} < n\epsilon, \quad (12)$$

while the hoop conjecture requires $E \sim n\epsilon < R$. Combined, we obtain

$$\mathcal{R} < R \sim V^{1/3}. \quad (13)$$

It is interesting to compare this bound to the rate of information processing performed by nature in the evolution of physical systems. At first glance, there appears to be a problem since one typically assumes the number of degrees of freedom in a region is proportional to V (is extensive). Then, the amount of information processing necessary to evolve such a system in time grows much faster than our bound (13) as V increases. Recall that for n degrees of freedom (for simplicity, qubits), the dimension of Hilbert space H is $N = \dim H = 2^n$ and the entropy is $S = \ln N \sim n$. In the extensive case, $n \sim S \sim V$.

However, as noted in the previous section, gravity also constrains the maximum information content (entropy S) of a region of space. 't Hooft [25] showed that if one excludes states from the Hilbert space whose energies are so large that they would have already caused gravitational collapse, one obtains $S = \ln N < A^{3/4}$, where N is the number of degrees of freedom and A the surface area.

Given this result we can calculate the maximum rate of information processing necessary to simulate any physical system of volume V which is not a black hole. The rate \mathcal{R}

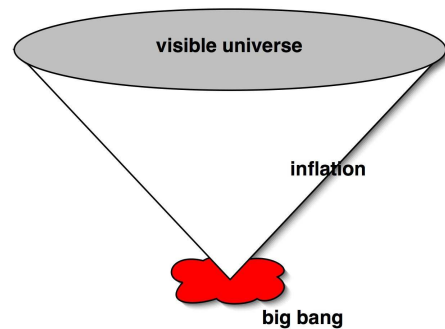


FIG. 4: The visible universe is exponentially larger than the initial region from which it evolved.

is given by the number of degrees of freedom $S \sim R^{3/2}$ times the maximal ML rate $T \sim R^{-1/2}$. This yields $\mathcal{R} \sim R$ as in our bound (13).

Finally, we note that black holes themselves appear to saturate our bound. If we take the black hole entropy to be $S \sim A \sim R^2$, and the typical energy of its modes to be the Hawking temperature $T_H \sim R^{-1}$, we again obtain $\mathcal{R} \sim R$.

VI. HOW MUCH INFORMATION IN THE UNIVERSE?

In this final section we ask how much information is necessary to specify the current state of the universe, and where did it come from?

There is convincing observational evidence for the big bang model of cosmology, and specifically for the fact that the universe is and has been expanding. In a radiation-dominated universe, the FRW scale factor grows as $R(t) \sim t^{1/2}$, where t is the comoving cosmological time. From this, it is clear that our universe evolved from a much smaller volume at early times. Indeed, in inflationary cosmology (Fig. 4) the visible universe results from an initial patch which is exponentially smaller than our current horizon volume. The corresponding ratio of entropies is similarly gigantic, meaning that there is much more information in the universe today than in the small primordial patch from which it originated. Therefore, the set of possible early universe initial conditions is much, much smaller than the set of possible late time universes. A mapping between all the detailed rearrangements or modifications of the universe today and the set of possible initial data is many to one, not one to one [33].

Thus, the richness and variability of the universe we inhabit cannot be attributed to the range of initial conditions. The fact that I am typing this on a sunny day, or that our planet has a single moon, or that the books on my office shelves have their current arrangement, was not determined by big bang initial data.

How, then, do the richness and variability of our world

arise? The answer is quantum randomness – the randomness inherent in measurements of quantum outcomes.

Imagine an ensemble Ψ of n qubits, each prepared in an identical state ψ . Now imagine that each qubit is measured, with a resulting spin up (+) or spin down (−) result. There are 2^n possible records, or histories, of this measurement. This is an exponentially large set of outcomes; among them are all possible n -bit strings, including every n -bit work of literature it is possible to write! Although the initial state Ψ contained very little information (essentially, only a single qubit of information, since each spin is in an identical state), n bits of classical information are required to specify *which* of the 2^n outcomes is observed in a particular universe. For $n \rightarrow \infty$ the set of possible records is arbitrarily rich and varied despite the simplicity of initial state Ψ .

In the same way, given an initial quantum state Ψ describing the primordial patch of the big bang from which our horizon volume evolved, one must still know the outcomes of a large number of quantum measurements in order to specify the particulars of the universe today. From a many worlds perspective, one must specify all the decoherent outcomes to indicate a particular branch of the wavefunction – a staggering amount of information. Equivalently, from the traditional Copenhagen perspec-

tive, each quantum measurement injects a bit (or more) of truly random information into our universe, and this randomness accounts for its variability.

The most familiar cosmological quantum randomness comes from fluctuations of the inflaton field, which determine the spectrum of primordial energy density fluctuations. It is these density fluctuations that determine the locations of galaxies, stars and planets today. However, from entropic or information theoretic considerations we readily deduce that essentially *every* detailed aspect of our universe (beyond the fundamental Lagrangian and some general features of our spacetime and its contents) is a consequence of quantum fluctuations!

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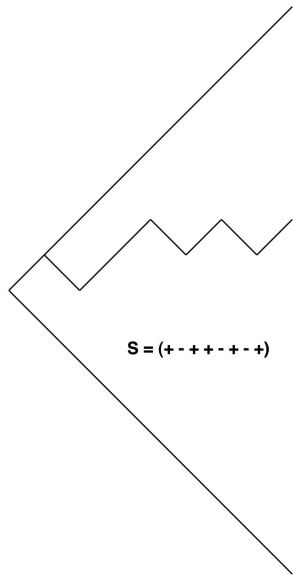


FIG. 5: Each path represents a set of outcomes S from the measurement of n qubits. To specify a particular path requires n bits of classical information.

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[32] We identify n individual components for later comparison with entropy. The ML theorem could, of course, be applied to the entire device as well.

[33] Of course, it is possible that not all seemingly innocuous rearrangements of our universe are possible. Perhaps only an infinitesimal subset are possible future evolutions of the initial data – this would be the case in a purely classical or deterministic universe. The situation would be analogous to a pseudo-random number generator: from a finite seed of limited entropy, it produces a much larger, seemingly random and aperiodic sequence of output. However, varying over all possible seeds does not produce all possible output strings, since the process is deterministic and the seed entropy is small.